Wahlaufgaben

Aufgabe 2011 W1a:

5,5 P

Im Dreieck ABC gilt:

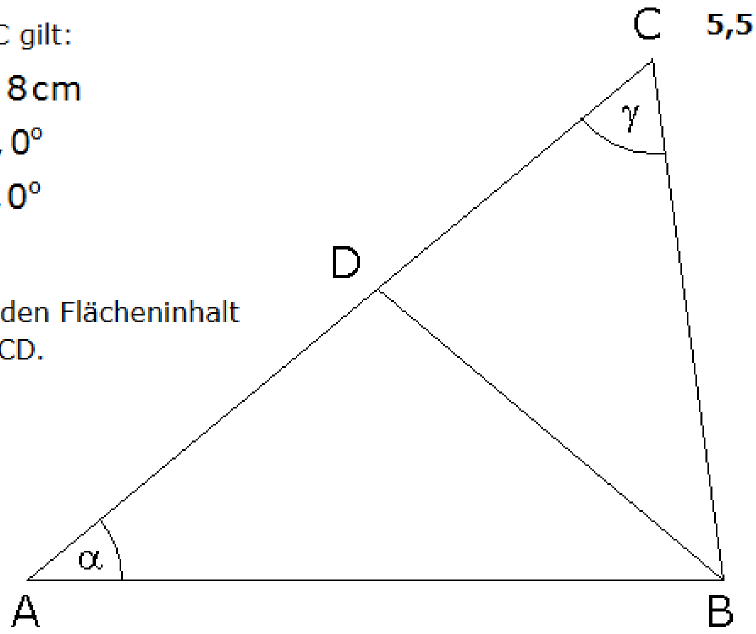
$\overline{AB} = 10,8\,\text{cm}$

$\alpha = 40,0^\circ$

$\gamma = 58,0^\circ$

$\overline{AD} = \overline{BD}$

Berechnen Sie den Flächeninhalt des Dreiecks BCD.



Strategie 2011 W1a:

Gegeben:

Allgemeines Dreieck ABC

$\overline{AB} = 10,8\,\text{cm}$

$\alpha = 40,0^\circ$

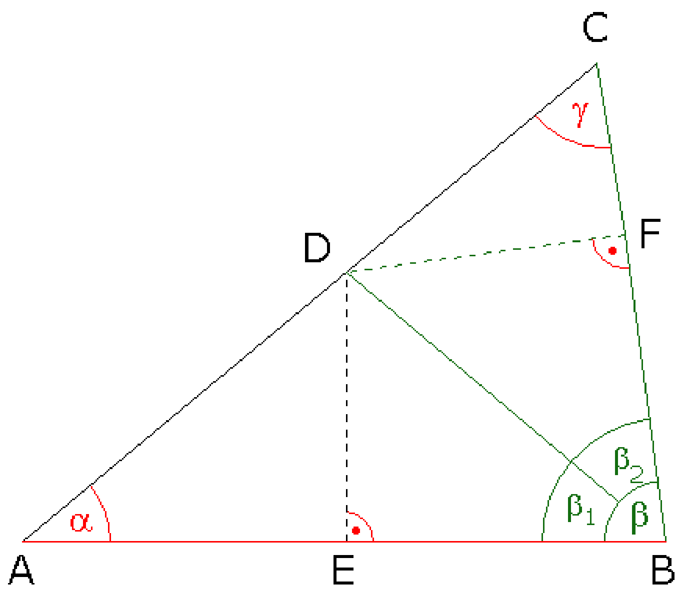
$\gamma = 58,0^\circ$

$\overline{AD} = \overline{BD}$

Gesucht:

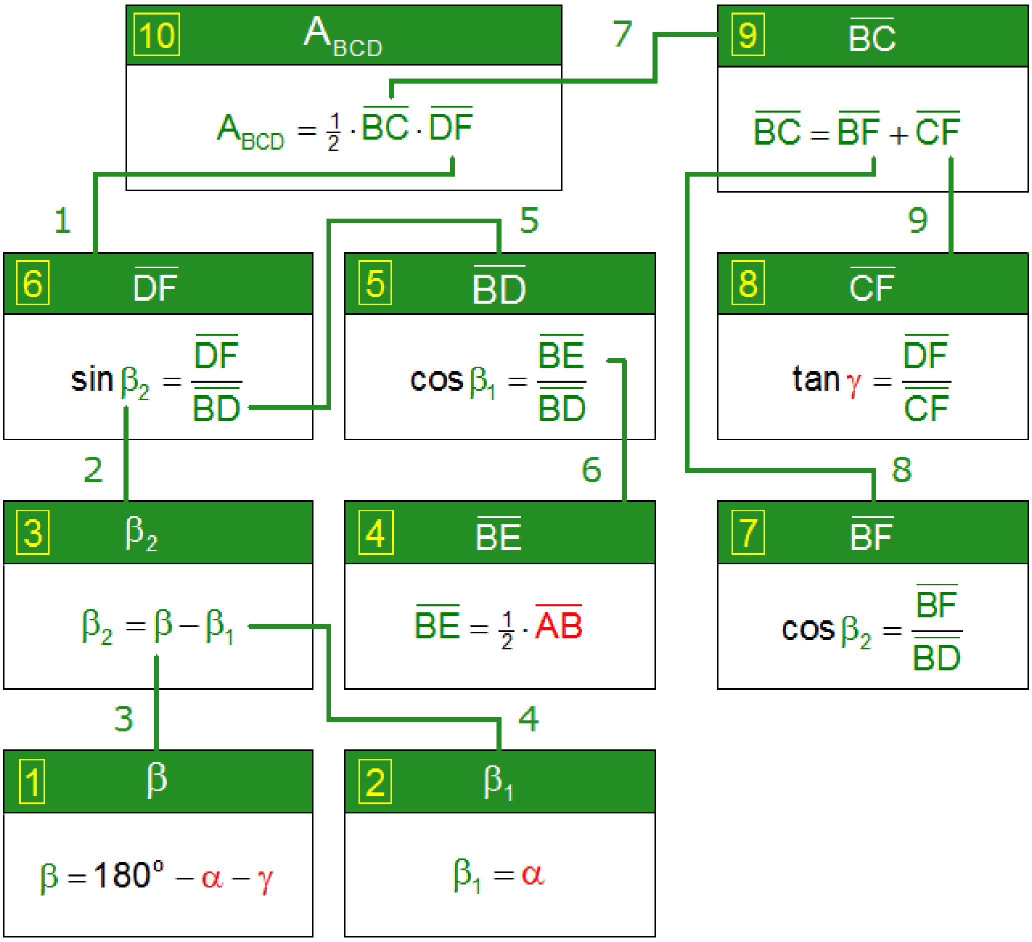
A_{BCD}

Skizze:



Strategie 2011 W1a:

Struktogramm:



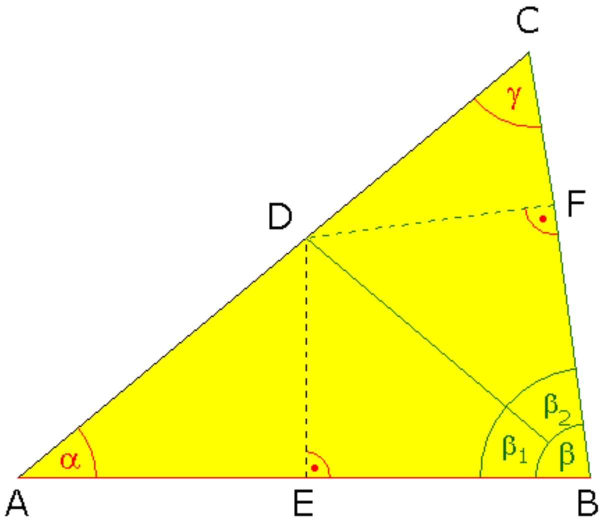
Lösung 2011 W1a:

1. Berechnung des Winkels β:

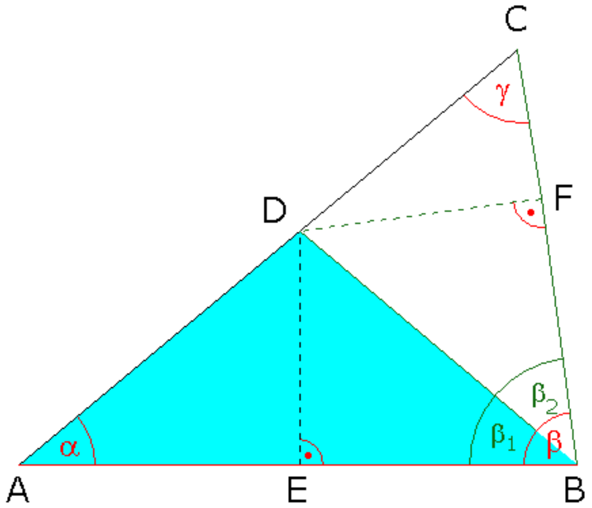
$\alpha + \beta + \gamma = 180^\circ$ Winkelsumme im gelben Dreieck ABC

$40^\circ + \beta + 58^\circ = 180^\circ \quad | -40^\circ - 58^\circ$

$\underline{\beta = 82^\circ}$



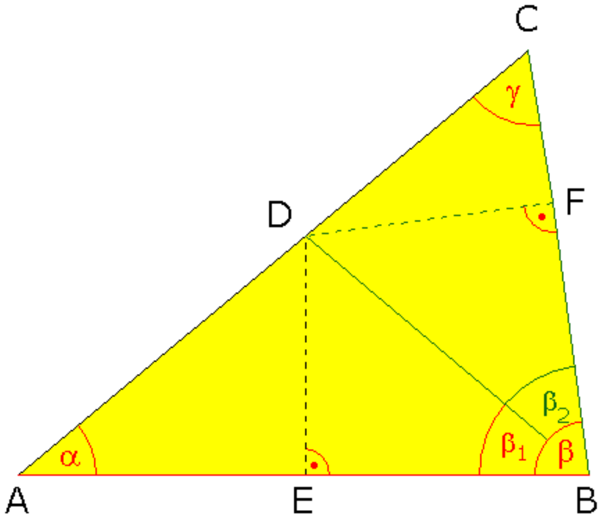
Lösung 2011 W1a:



2. Bestimmung des Winkels β_1:

$\beta_1 = \alpha$ $\overline{AD} = \overline{BD} \Rightarrow \Delta_{ABD}$ ist gleichschenklig,
d. h. Basiswinkel sind gleich groß.

$\underline{\beta_1 = 40^\circ}$

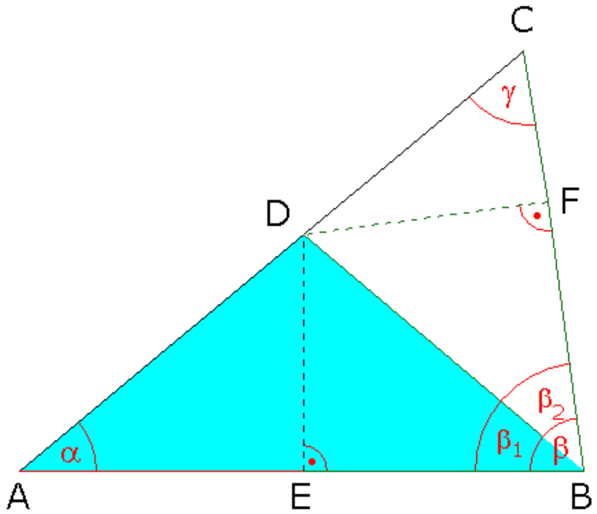


3. Berechnung des Winkels β_2:

$\beta_1 + \beta_2 = \beta$

$40^\circ + \beta_2 = 82^\circ \quad | -40^\circ$

$\underline{\beta_2 = 42^\circ}$



4. Berechnung der Strecke $\overline{BE}$:

$\overline{BE} = \frac{1}{2} \cdot \overline{AB}$ $\overline{AD} = \overline{BD} \Rightarrow \Delta_{ABD}$ ist gleichschenklig,
d. h. E halbiert $\overline{AB}$.

$\overline{BE} = \frac{1}{2} \cdot 10,8$

$\underline{\overline{BE} = 5,4\,\text{cm}}$

Lösung 2011 W1a:

5. Berechnung der Strecke $\overline{BD}$:

$$\cos\beta_1 = \frac{\text{Ankathete}}{\text{Hypotenuse}} = \frac{\overline{BE}}{\overline{BD}}$$

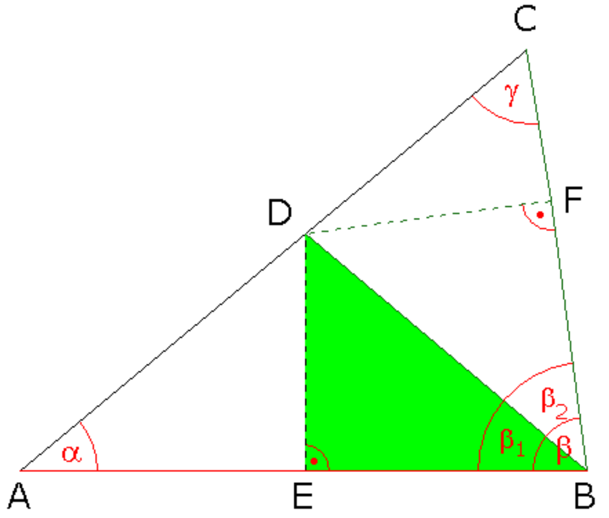
Kosinusfunktion im rechtwinkligen grünen Teildreieck BDE

$$\cos 40^\circ = \frac{5,4}{\overline{BD}}$$

$$0,7660 = \frac{5,4}{\overline{BD}} \qquad \Big|\cdot \overline{BD}$$

$$\overline{BD}\cdot 0,7660 = 5,4 \qquad \Big|: 0,7660$$

$$\underline{\overline{BD} = 7,05\,\text{cm}}$$



6. Berechnung der Strecke $\overline{DF}$:

$$\sin\beta_2 = \frac{\text{Gegenkathete}}{\text{Hypotenuse}} = \frac{\overline{DF}}{\overline{BD}}$$

Sinusfunktion im rechtwinkligen orangefarbenen Teildreieck BFD

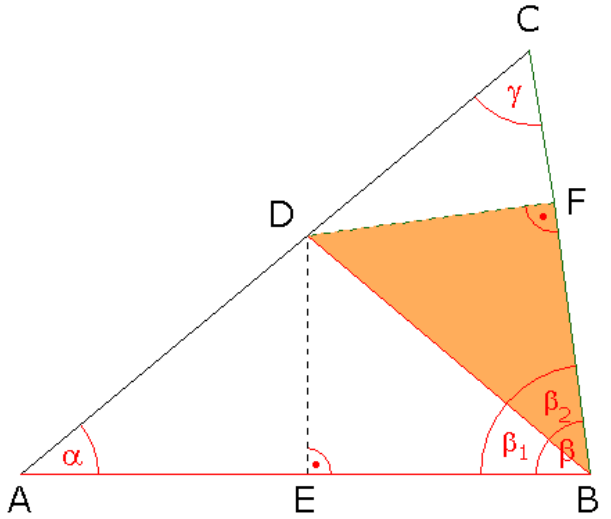
$$\sin 42^\circ = \frac{\overline{DF}}{7,05}$$

$$0,6691 = \frac{\overline{DF}}{7,05}$$

Seiten tauschen

$$\frac{\overline{DF}}{7,05} = 0,6691 \qquad \Big|\cdot 7,05$$

$$\underline{\overline{DF} = 4,72\,\text{cm}}$$



7. Berechnung der Strecke $\overline{BF}$:

$$\cos\beta_2 = \frac{\text{Ankathete}}{\text{Hypotenuse}} = \frac{\overline{BF}}{\overline{BD}}$$

Kosinusfunktion im rechtwinkligen orangefarbenen Teildreieck BFD

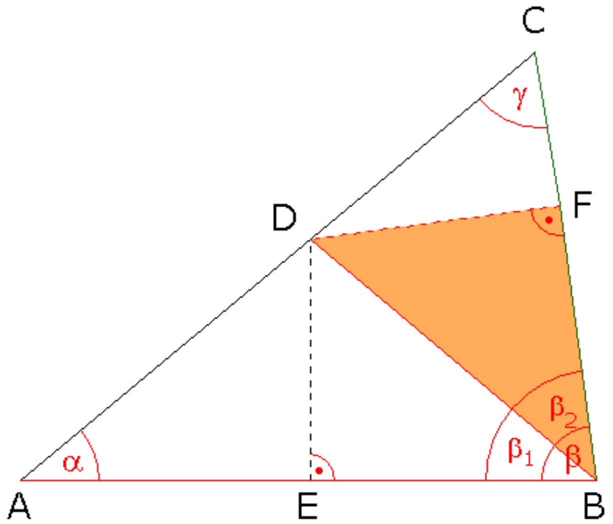
$$\cos 42^\circ = \frac{\overline{BF}}{7,05}$$

$$0,7431 = \frac{\overline{BF}}{7,05}$$

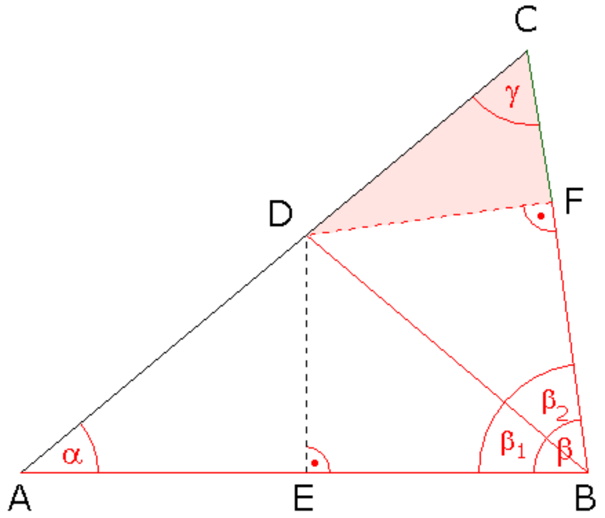
Seiten tauschen

$$\frac{\overline{BF}}{7,05} = 0,7431 \qquad \Big|\cdot 7,05$$

$$\underline{\overline{BF} = 5,24\,\text{cm}}$$



Lösung 2011 W1a:



8. Berechnung der Strecke $\overline{CF}$:

$$\tan\gamma = \frac{\text{Gegenkathete}}{\text{Ankathete}} = \frac{\overline{DF}}{\overline{CF}}$$

Tangensfunktion im rechtwinkligen magentefarbenen Teildreieck CDF

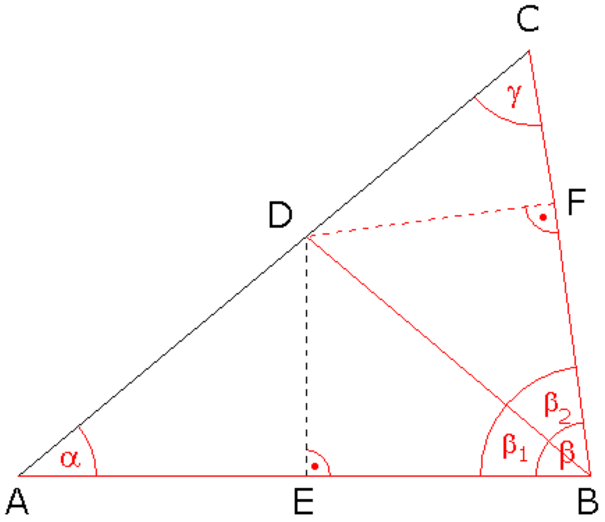
$$\tan 58^\circ = \frac{4,72}{\overline{CF}}$$

$$1,6 = \frac{4,72}{\overline{CF}} \qquad \Big| \cdot \overline{CF}$$

$$\overline{CF} \cdot 1,6 = 4,72 \qquad \Big| : 1,6$$

$$\underline{\overline{CF} = 2,95\,\text{cm}}$$

9. Berechnung der Strecke $\overline{BC}$:

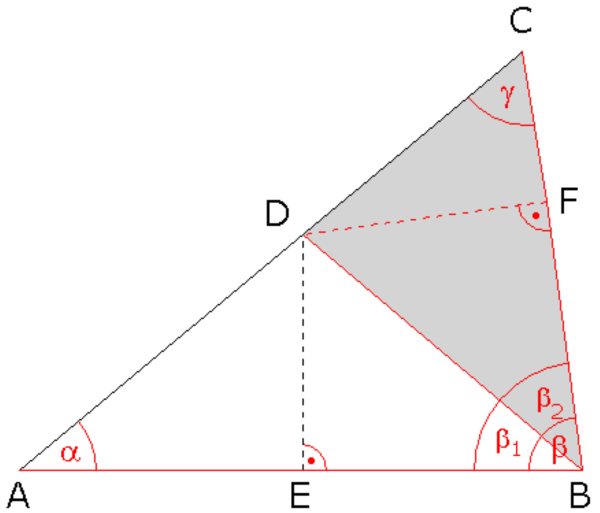


$$\overline{BC} = \overline{BF} + \overline{CF}$$

$$\overline{BC} = 5,24 + 2,95$$

$$\underline{\overline{BC} = 8,19\,\text{cm}}$$

10. Berechnung der Dreiecksfläche A_{BCD}:



$$A_{BCD} = \frac{1}{2} \cdot g \cdot h = \frac{1}{2} \cdot \overline{BC} \cdot \overline{DF}$$

Flächenformel allgemeines Dreieck

$$A_{BCD} = \frac{1}{2} \cdot 8,19 \cdot 4,72$$

$$\underline{\underline{A_{BCD} = 19,3\,\text{cm}^2}}$$