Wahlaufgaben

Aufgabe 2003 W4a:

4,5 P

Vom gleichschenkligen Trapez ABCD sind gegeben:

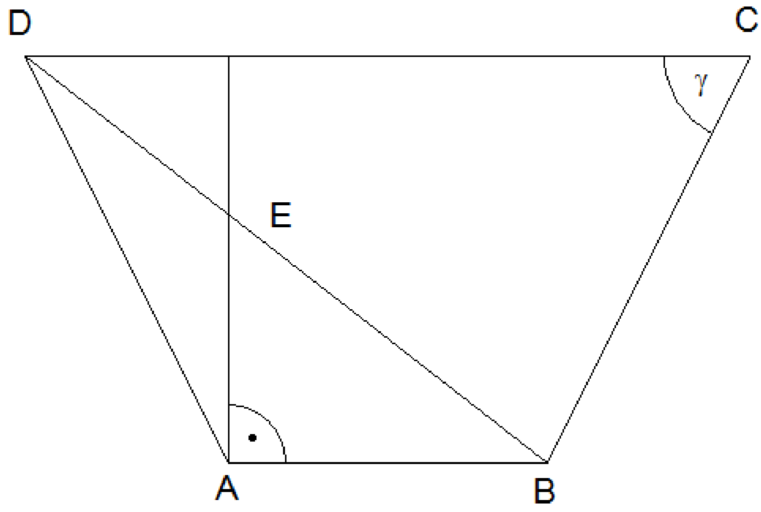
$\overline{AB} = 5,6\,\text{cm}$

$\overline{AD} = 7,8\,\text{cm}$

$\gamma = 64,2°$

Berechnen Sie die Länge $\overline{AE}$.

Welchen Abstand hat E von $\overline{BC}$?



Strategie 2003 W4a:

Gegeben:

Gleichschenkliges Trapez

$\overline{AB} = 5,6\,\text{cm}$

$\overline{AD} = 7,8\,\text{cm}$

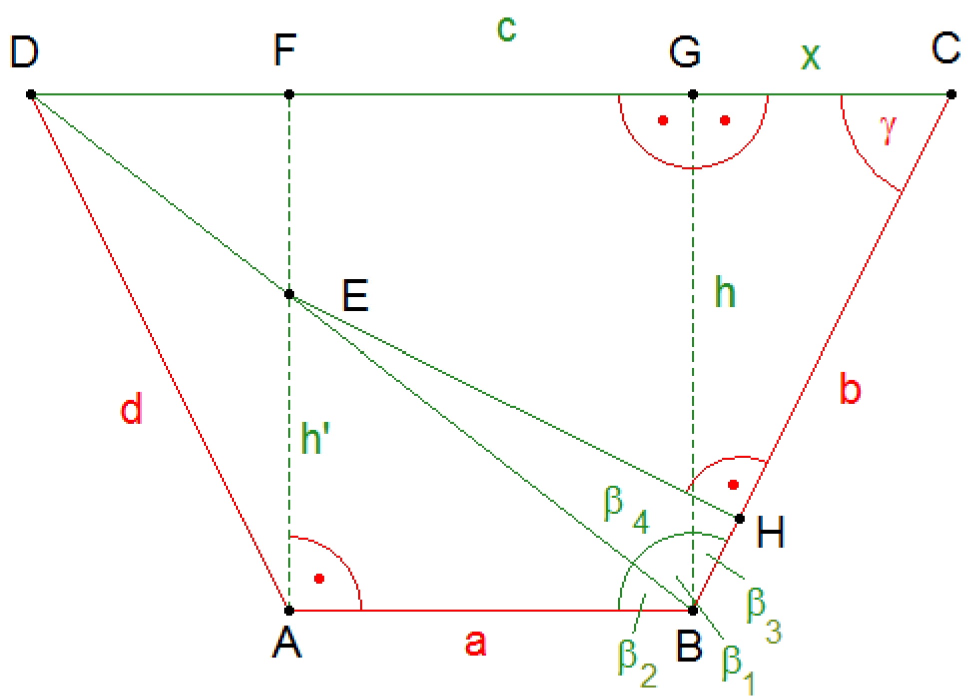
$\gamma = 64,2°$

Gesucht:

$\overline{AE}$

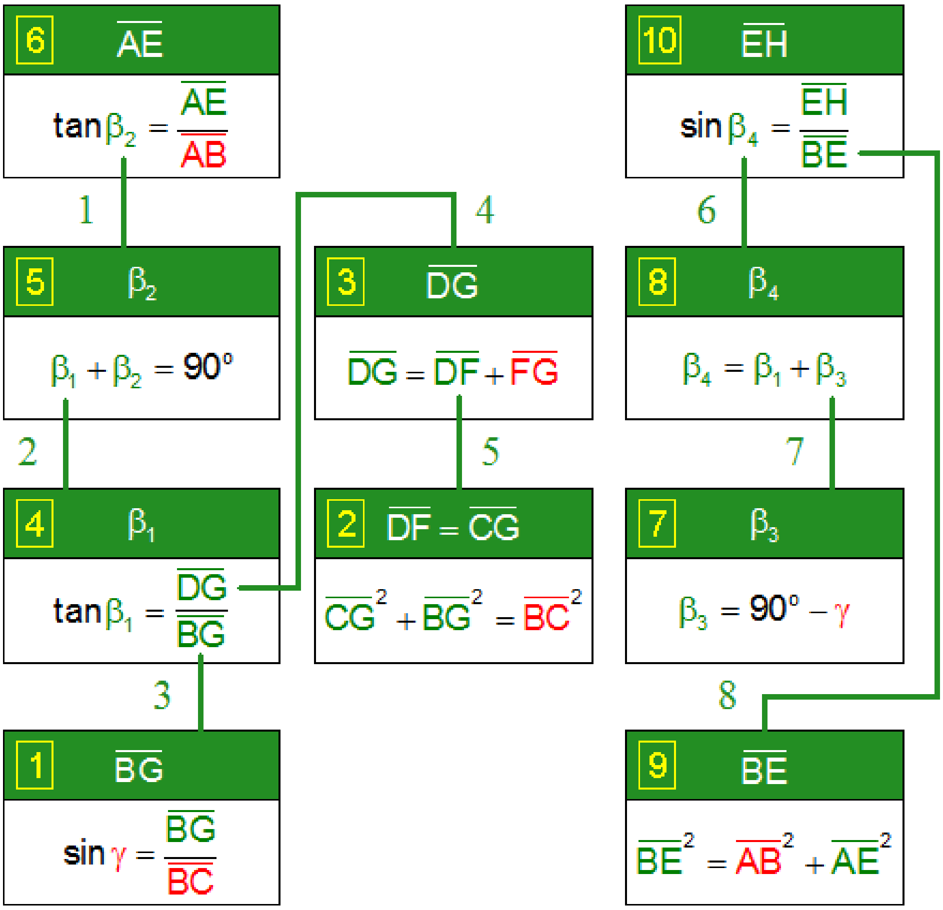
$\overline{EH}$

Skizze:



Strategie 2003 W4a:

Struktogramm:



Lösung 2003 W4a:

1. Berechnung der Trapezhöhe $\overline{BG} = h$:

$$\sin\gamma = \frac{\text{Gegenkathete}}{\text{Hypotenuse}} = \frac{\overline{BG}}{\overline{BC}} = \frac{h}{b}$$ Sinusfunktion im rechtwinkligen gelben Teildreieck

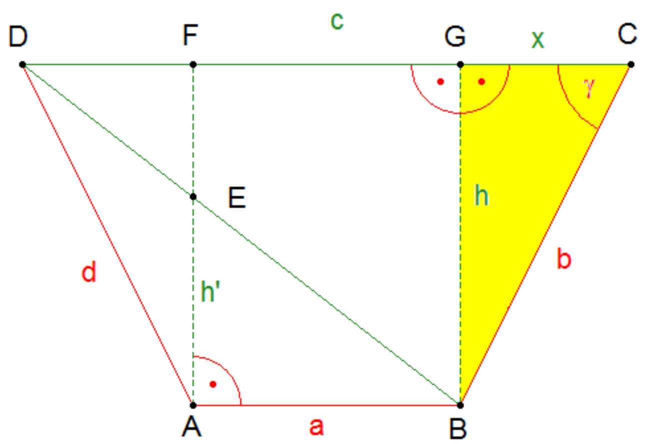
$$\sin 64{,}2° = \frac{h}{7{,}8}$$

$$0{,}9003 = \frac{h}{7{,}8}$$ Seiten tauschen

$$\frac{h}{7{,}8} = 0{,}9003 \quad | \cdot 7{,}8$$

$$\underline{h = 7{,}02\,\text{cm}}$$

$$\underline{\overline{BG} = 7{,}02\,\text{cm}}$$



Lösung 2003 W4a:

2. Berechnung der Strecke $\overline{CG} = x$:

$$\overline{CG}^2 + \overline{BG}^2 = \overline{BC}^2$$

Pythagoras im rechtwinkligen gelben Teildreieck BCG

$$x^2 + h^2 = b^2$$

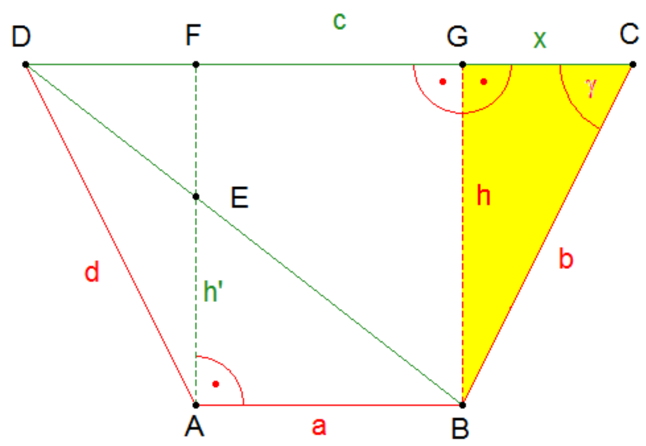
$$x^2 + 7{,}02^2 = 7{,}8^2$$

$$x^2 + 49{,}28 = 60{,}48 \quad \left| -49{,}28 \right.$$

$$x^2 = 11{,}56 \quad \left| \sqrt{} \right.$$

$$\underline{x = 3{,}40\,\text{cm}}$$

$$\underline{\overline{CG} = 3{,}40\,\text{cm}}$$



3. Berechnung der Strecke $\overline{DG}$:

$$\overline{DG} = \overline{DF} + \overline{FG}$$

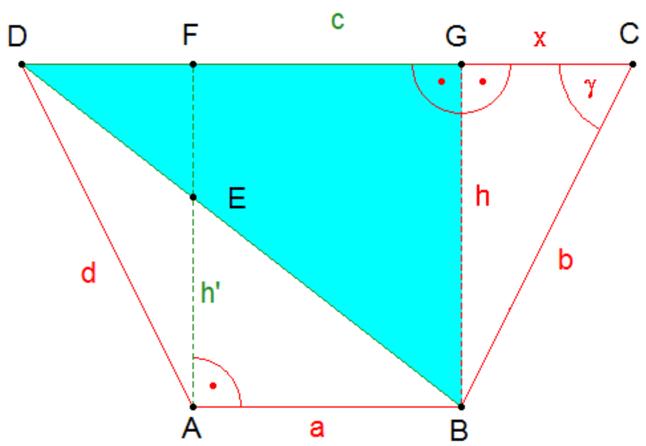
$$\overline{DG} = x + a$$

$\overline{DF} = x$; da gleichschenkliges Trapez

$\overline{FG} = \overline{AB} = a$

$$\overline{DG} = 3{,}4 + 5{,}6$$

$$\underline{\overline{DG} = 9{,}0\,\text{cm}}$$



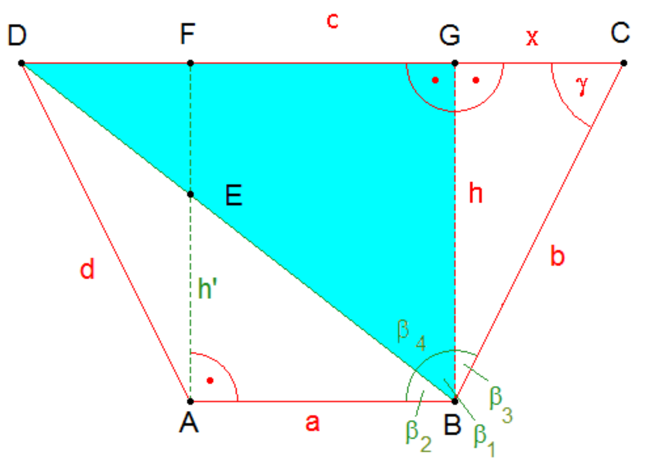
4. Berechnung des Winkels β_1:

$$\tan\beta_1 = \frac{\text{Gegenkathete}}{\text{Ankathete}} = \frac{\overline{DG}}{\overline{BG}} = \frac{\overline{DG}}{h}$$

Tangensfunktion im rechtwinkligen blauen Teildreieck

$$\tan\beta_1 = \frac{9}{7{,}02} = 1{,}2821$$

$$\underline{\beta_1 = 52{,}0^\circ}$$

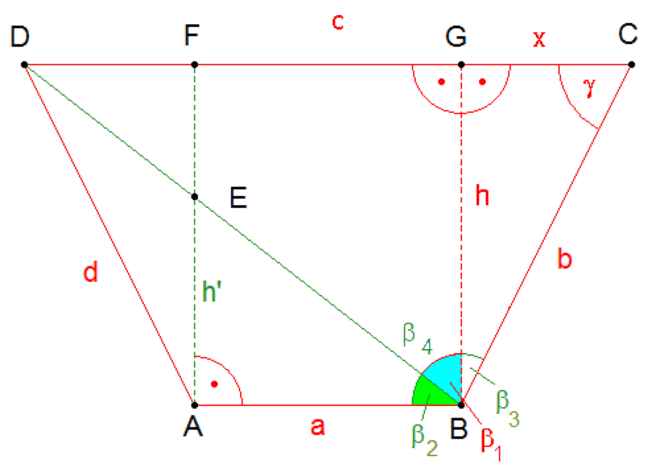


5. Berechnung des Winkels β_2:

$$\beta_1 + \beta_2 = 90^\circ$$

$$52^\circ + \beta_2 = 90^\circ \quad \left| -52^\circ \right.$$

$$\underline{\beta_2 = 38^\circ}$$



Lösung 2003 W4a:

6. Berechnung der Strecke $\overline{AE}$:

$$\tan\beta_2 = \frac{\text{Gegenkathete}}{\text{Ankathete}} = \frac{\overline{AE}}{\overline{AB}} = \frac{\overline{AE}}{a}$$

Tangensfunktion im rechtwinkligen grünen Teildreieck

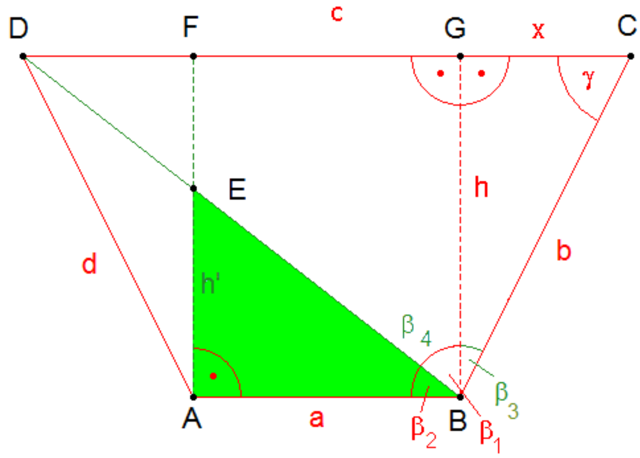
$$\tan 38^\circ = \frac{\overline{AE}}{5,6}$$

$$0,7813 = \frac{\overline{AE}}{5,6}$$

Seiten tauschen

$$\frac{\overline{AE}}{5,6} = 0,7813 \qquad |\cdot 5,6$$

$$\underline{\underline{\overline{AE} = 4,38\,\text{cm}}}$$

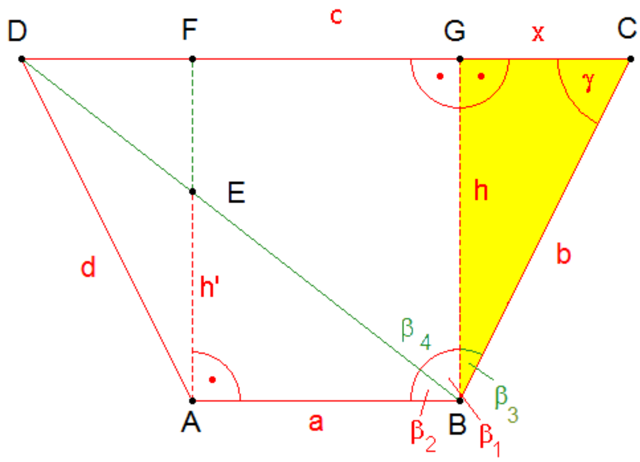


7. Berechnung des Winkels β_3:

$$\beta_3 = 90^\circ - \gamma$$

$$\beta_3 = 90^\circ - 64,2^\circ$$

$$\underline{\beta_3 = 25,8^\circ}$$

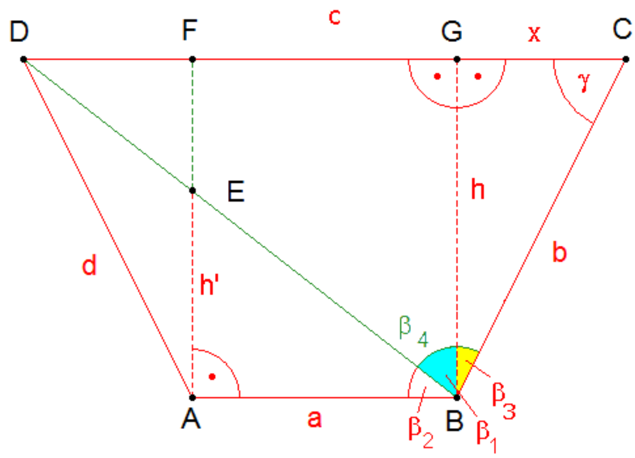


8. Berechnung des Winkels β_4:

$$\beta_4 = \beta_1 + \beta_3$$

$$\beta_4 = 52^\circ + 25,8^\circ$$

$$\underline{\beta_4 = 77,8^\circ}$$



9. Berechnung der Strecke $\overline{BE}$:

$$\overline{BE}^2 = \overline{AB}^2 + \overline{AE}^2 = a^2 + h'^2$$

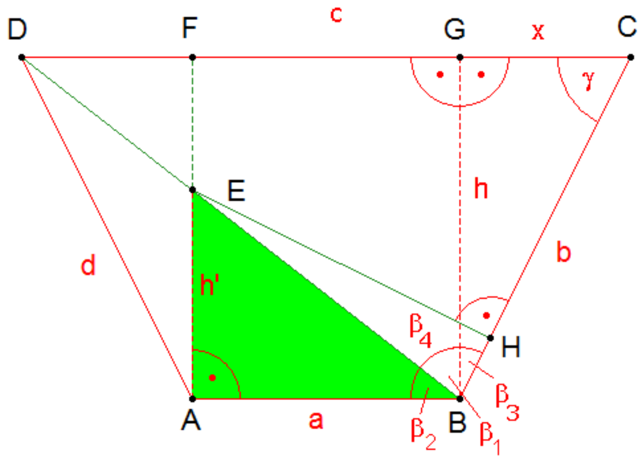
Pythagoras im rechtwinkligen grünen Teildreieck

$$\overline{BE}^2 = 5,6^2 + 4,38^2$$

$$\overline{BE}^2 = 31,36 + 19,18$$

$$\overline{BE}^2 = 50,54 \qquad |\sqrt{}$$

$$\underline{\overline{BE} = 7,11\,\text{cm}}$$



Lösung 2003 W4a:

10. Berechnung der Strecke $\overline{EH}$:

$$\sin\beta_4 = \frac{\text{Gegenkathete}}{\text{Hypotenuse}} = \frac{\overline{EH}}{\overline{BE}}$$ Sinusfunktion im rechtwinkligen orangefarbenen Teildreieck

$$\sin 77{,}8^\circ = \frac{\overline{EH}}{7{,}11}$$

$$0{,}9774 = \frac{\overline{EH}}{7{,}11}$$ Seiten tauschen

$$\frac{\overline{EH}}{7{,}11} = 0{,}9774 \qquad \left|\cdot 7{,}11\right.$$

$$\underline{\underline{\overline{EH} = 6{,}95\,\text{cm}}}$$